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CRITICAL CHANGE OF ION CONDUCTANCE AND SELF-SUSTAINED POTENTIAL OSCILLATION IN ARTIFICIAL MEMBRANE SYSTEMS

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A change in conductance of an artificial membrane at a threshold concentration of environmental salt solution was interpreted in terms of a change in adsorptive reaction rate on the interface which was derived from an autocatalytic reaction model. The model also accounted for a self-sustained potential oscillation which was observed when the salt concentration of one side of the membrane was higher than the threshold mentioned above and that of the other side lower.

1. Introduction

Excitations and self-sustained oscillations in biological membranes have been extensively investigated during the last half-century. Similar phenomena have also been observed in various kinds of artificial membrane systems. Teorell [1] demonstrated a self-sustained oscillation in a system consisting of salt solutions with different concentrations separated by a porous glass membrane, through which a constant current was passed.

A simpler system consisting of oil and water phases has been reported to exhibit electrochemical oscillations. Dupeyrat and Nakache [2,3] observed quasi-periodic variations of the interfacial tension and electric potential in an oil/water interface system composed of solutions of hexadecyltrimethylammonium chloride in water and picric acid in nitroethane or nitrobenzene. They proposed a mechanism for a chemical oscillating reaction which would make periodic variations of the two species in the two minor zones of the bulk phase near the interface and lead to periodic deformation of the interface. Yoshikawa and Matsubara [4] reported self-sustained oscillations in a two-phase system of a solution of picric acid in 2-nitropropane and an aqueous solution of hexadecyltrimethylammonium bromide (CTAB). Another example of an oil/water system is an oil layer of nitrobenzene containing picric acid, imposed between two aqueous phases, one containing CTAB and alcohol and the other containing sucrose [5].

A more controllable system is a Millipore filter impregnated with dioleyl phosphate (DOPH) [6–8]. This artificial membrane is reported to have the following characteristics:

(1) The DOPH-Millipore membrane immersed in a KCl solution displays a rapid increase in electric capacitance [6] and electric conductance [7] at a certain critical salt concentration, c when the concentration is increased.

(2) When the salt concentration of one side of the membrane is higher than c and that of the other side lower, self-sustained potential spikes with a long period are observed across the membrane [6,8].

(3) In addition to the concentration difference mentioned above, a pressure difference is applied

across the membrane (higher pressure on the more concentrated side) and a d.c. electric current is passed from the more concentrated to the dilute side through the membrane. The spike-like potential then changes into a continuous oscillation with a short period [6,9].

Toko et al. [8,9] proposed a model for the oscillatory phenomena of this system which was based on a phase transition due to the external salt concentration accompanied by a conformational change in the DOPH molecular assembly [6] among droplets, spherical micelles and multilayers. The self-sustained oscillation was accounted for by an iteration of accumulation and release of salt concentration in pores of the filter coupled with the phase transition.

In this report, we present a model of an autocatalytic mechanism for the adsorptive reaction at the gate of each pore in the membrane surface, leading to a critical change in conductance of the membrane and a self-sustained oscillation of the electric potential across the membrane.

2. Autocatalytic model for ion-adsorptive reaction

The proposed model for the adsorption process consists of the following steps.

(i) The ions in the main bulk of the solution move to the immediate vicinity of the membrane by way of a diffusion process. We denote the X ion (e.g., K^+) on one side of the membrane by X_A and the same ion on the other side by X_B.

(ii) The first step of adsorption of the X_A ion by the membrane substance M (e.g., DOPH) takes place at the gate of the pore in a way expressed by

$$M + X_A \underset{K_1'}{\overset{K_1}{\rightleftharpoons}} MX. \qquad (1)$$

Fig. 1 shows a schematic diagram of the potential energy for the adsorption and desorption processes of the X ion by the membrane substance M in a pore channel.

(iii) Here we propose another process in which the intermediate product MX serves as a catalyst for the adsorption of X_A,

$$M + X_A + 2MX \underset{K_2'}{\overset{K_2}{\rightleftharpoons}} 3MX. \qquad (2)$$

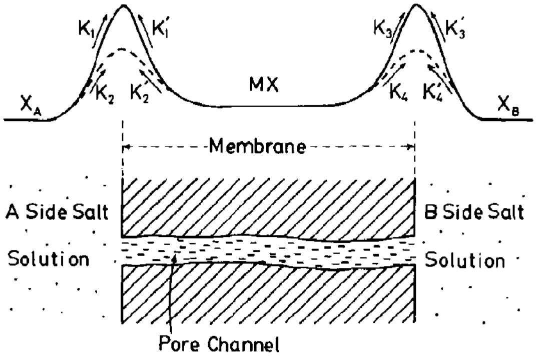

Fig. 1. Schematic diagram of the potential energy for the adsorption of the ion (X) from the solution and the desorption of the intermediate product (MX) from the pore channel of the membrane.

The physical meaning of this process is that X ions, once adsorbed in the channel, form a pair and cause further adsorption of another X ion around them. We have no direct evidence for this mechanism in the adsorption process of the ions on the membrane surface, but the same situation takes place in the nucleation process of crystallization of metal and compound from their solutions. As an alternative mechanism for the catalytic reaction (eq. 2), a conformational change of membrane substances [9] may be considered. If we assume that a micelle is formed only when a critical number of intermediate products MX aggregate, then formation of a micelle would be expressed as follows,

$$M + X_A + (n-1)MX \rightleftharpoons nMX. \qquad (3)$$

Eq. 2 is a simple case for $n = 3$.

(iv) The intermediate products MX move in the pore channel by a diffusion process.

(v) On the other side of the membrane, the same processes as mentioned in items ii and iii take place as follows;

$$MX \underset{K_3'}{\overset{K_3}{\rightleftharpoons}} M + X_B \qquad (4)$$

and

$$3MX \underset{K_4'}{\overset{K_4}{\rightleftharpoons}} M + X_B + 2MX. \qquad (5)$$

We denote the concentrations of X ions in the vicinity of the membrane on the A and B sides simply as X_A and X_B, respectively and those in the main bulk of the solutions as X_{A0} and X_{B0}. Further, the concentrations of the intermediate products MX on the A and B sides of the channel are denoted by Y_A and Y_B, respectively. The rate equations for Y_A, Y_B, X_A and X_B are then given in the form including reactions and diffusion,

$$\frac{dY_A}{dt} = K_1 M X_A - K_1' Y_A + K_2 M X_A Y_A^2 - K_2' Y_A^3 + D_M(Y_B - Y_A) \tag{6}$$

$$\frac{dY_B}{dt} = K_3' M X_B - K_3 Y_B + K_4' M X_B Y_B^2 - K_4 Y_B^3 + D_M(Y_A - Y_B) \tag{7}$$

$$\frac{dX_A}{dt} = -K_1 M X_A + K_1' Y_A - K_2 M X_A Y_A^2 + K_2' Y_A^3 + D_A(X_{A0} - X_A) \tag{8}$$

$$\frac{dX_B}{dt} = -K_3' M X_B + K_3 Y_B - K_4' M X_B Y_B^2 + K_4 Y_B^3 + D_B(X_{B0} - X_B). \tag{9}$$

where D_M is the diffusion coefficient of the intermediate product in the channel and D_A and D_B those of X ions in the solutions of the A and B sides, respectively. The diffusion coefficient of the ions is basically determined by a scattering process due to solvent molecules, but at high ionic concentrations, coulomb scattering by other solute ions of the same kind and/or of the opposite charge has an effect on the diffusion process. Thus, the diffusion coefficient on the highly concentrated side may be less than that on the other side with lower concentration. The first four terms on the right-hand side of each of eqs. 6–9 represent the change in concentrations due to the adsorptive and desorptive reactions at the channel gate on the membrane surface, while the fifth term represents the change due to the diffusion process.

When the ion concentrations in the solutions on the two sides are equal, the membrane is symmetric and we can put

$$X_A = X_B = X, \; Y_A = Y_B = Y \tag{10}$$

$$K_1' = K_3, \; K_2' = K_4, \; K_3' = K_1, \; K_4' = K_2. \tag{11}$$

In this case, eqs. 6 and 7 are reduced to an identical form

$$\frac{dY}{dt} = K_1 M X - K_3 Y + K_2 M X Y^2 - K_4 Y^3. \tag{12}$$

Let us plot dY/dt vs. Y curves for three typical values of ion concentration X in fig. 2. Here we take the kinetic constants as $K_1 = 0.1$, $K_2 = 2$, $K_3 = 2$ and $K_4 = 1.6$ and the concentration of the membrane substance as $M = 1.3$. In fig. 2, the curve for a small value of ion concentration ($X = 1.2$) intersects the abscissa only at a point P_1 which is an equilibrium stable point. With increasing ion concentration ($X = 1.5$), the curve intersects the abscissa at three points P_2, Q_2 and R_2, of which P_2 and R_2 are stable but Q_2 unstable. The curve for a further increased value of ion concentration ($X = 1.8$) intersects the abscissa again only at a stable point R_3. Therefore, the concentration of the intermediate substance Y will show a critical jump with increasing ion concentration X. If we increase X starting from a small value, Y at first takes small values corresponding to the points P_1 and P_2 and jumps to a large value corresponding to the point R_3 when

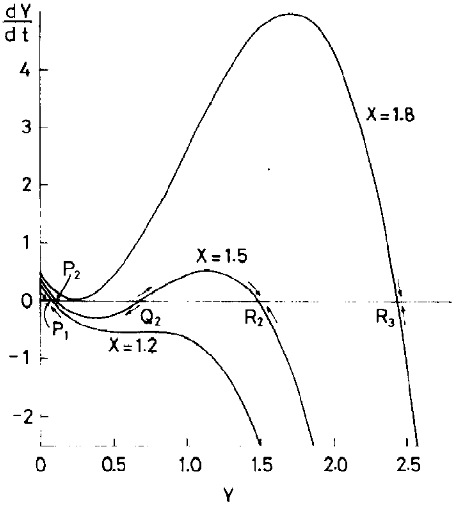

Fig. 2. dY/dt vs. Y curves given by eq. 12 for three values of X. Y is the concentration of the intermediate product in the pore channel of the membrane and X that of the ion in the environmental salt solution. Here the kinetic constants in eq. 12 are taken as $K_1 = 0.1$, $K_2 = 2$, $K_3 = 2$ and $K_4 = 1.6$.

the minimum point on the curve between P_2 and Q_2 separates from the abscissa. If X is decreased from a large value, Y at first takes rather large values corresponding to R_3 and R_2 and rapidly drops to a small value corresponding to P_1 when the maximum point on the curve between Q_2 and R_2 separates from the abscissa. This will lead to a change in the reaction rate and is an example of a phase transition in a nonequilibrium open system [10].

3. Critical change in conductance

The critical change in concentration of the intermediate substance Y reflects the conductance of the membrane. Suppose a very small voltage ΔV is applied across the membrane so as to make the A side positive and the B side negative. The applied voltage causes the forward kinetic constant K_i to change to

$$K_i \exp(\Delta V/RT) \div K_i(1+\Delta V/RT) \quad (13)$$

and the reverse kinetic constant K_i' to change to

$$K_i' \exp(-\Delta V/RT) \div K_i'(1-\Delta V/RT), \quad (14)$$

where R is the gas constant and T the absolute temperature. In eq. 14, the preexponential factors for the reverse kinetic constants are still expressed by (eq. 11), so that the specific form of each rate constant is given as follows,

$$K_1 \to K_1(1+\Delta V/RT), \quad K_1' \to K_3(1-\Delta V/RT) \quad (15)$$

$$K_2 \to K_2(1+\Delta V/RT), \quad K_2' \to K_4(1-\Delta V/RT) \quad (16)$$

$$K_3 \to K_3(1+\Delta V/RT), \quad K_3' \to K_1(1-\Delta V/RT) \quad (17)$$

$$K_4 \to K_4(1+\Delta V/RT), \quad K_4' \to K_2(1-\Delta V/RT). \quad (18)$$

The difference between the forward and reverse kinetic constants in the applied voltage lead to a net flow of ionic current across the membrane from the A side to the B side. The net current is given by the difference between the ionic flow into the channel from the A side solution and that from the B side, so that it is described in the form,

$$\Delta I = -\mathrm{d}X_A/\mathrm{d}t + \mathrm{d}X_B/\mathrm{d}t = 2(K_1MX + K_3Y + K_2MXY^2 + K_4Y^3)\Delta V/RT, \quad (19)$$

using eqs. 8, 9 and 15–18 and putting $X_{A0} = X_{B0} = X_A = X_B = X$ and $Y_A = Y_B = Y$. Thus, the conductance g is expressed by

$$g = \frac{\Delta I}{\Delta V} = 2(K_1MX + K_3Y + K_2MXY^2 + K_4Y^3)/RT. \quad (20)$$

For a given value of X, a steady-state value of Y is determined from eq. 12 and the conductance g is obtained by substituting X and Y into eq. 20. The result is plotted in fig. 3, where the parameters M, K_1, K_2, K_3 and K_4 have the same values as in fig. 2 and RT is taken as unity for simplicity. It can be seen that the conductance increases at $X = 1.78$ with increasing X and decreases at $X = 1.34$ with decreasing X.

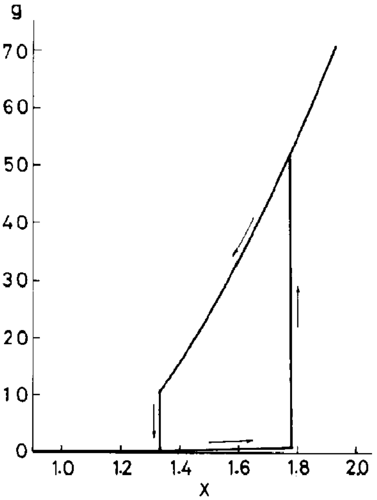

Fig. 3. Ionic conductance g of the membrane as a function of the ion concentration X in the solution. Parameters are the same as in fig. 2.

4. Self-sustained potential oscillation

When a DOPH-Millipore membrane is placed between two cells filled with aqueous solutions of low and high salt concentrations, a self-sustained potential of the pulse type with a long period is observed. In order to simulate this phenomenon, we have carried out a numerical calculation of the membrane potential as a function of time using eqs. 6–9 under the conditions that X_{A0} is larger and X_{B0} smaller than the threshold value described in section 3. We consider potential differences produced by an ion current due to adsorption and desorption of ions on the two interfaces between the membrane channel and the solutions. This potential change is assumed to cause a change in the rate constant through a change in the activation energy. If we take the direction of an ionic current flowing from the A to the B side to be forward, the currents through the A side interface by adsorption and the B side interface by desorption are given in the respective forms;

$$I_A = K_1 M X_A - K_1' Y_A + K_2 M X_A Y_A^2 - K_2' Y_A^3 \quad (21)$$

$$I_B = -K_3' M X_B + K_3 Y_B - K_4' M X_B Y_B^2 + K_4 Y_B^3 \quad (22)$$

which are the reaction terms in eqs. 8 and 9. These currents produce a temporal variation in potential difference and therefore cause a change in the rate constants as follows,

$$K_1 \to K_1 \exp(-I_A r/RT),$$
$$K_1' \to K_3 \exp(I_A r/RT) \quad (23)$$

$$K_2 \to K_2 \exp(-I_A r/RT),$$
$$K_2' \to K_4 \exp(I_A r/RT) \quad (24)$$

$$K_3 \to K_3 \exp(-I_B r/RT),$$
$$K_3' \to K_1 \exp(I_B r/RT) \quad (25)$$

$$K_4 \to K_4 \exp(-I_B r/RT),$$
$$K_4' \to K_2 \exp(I_B r/RT) \quad (26)$$

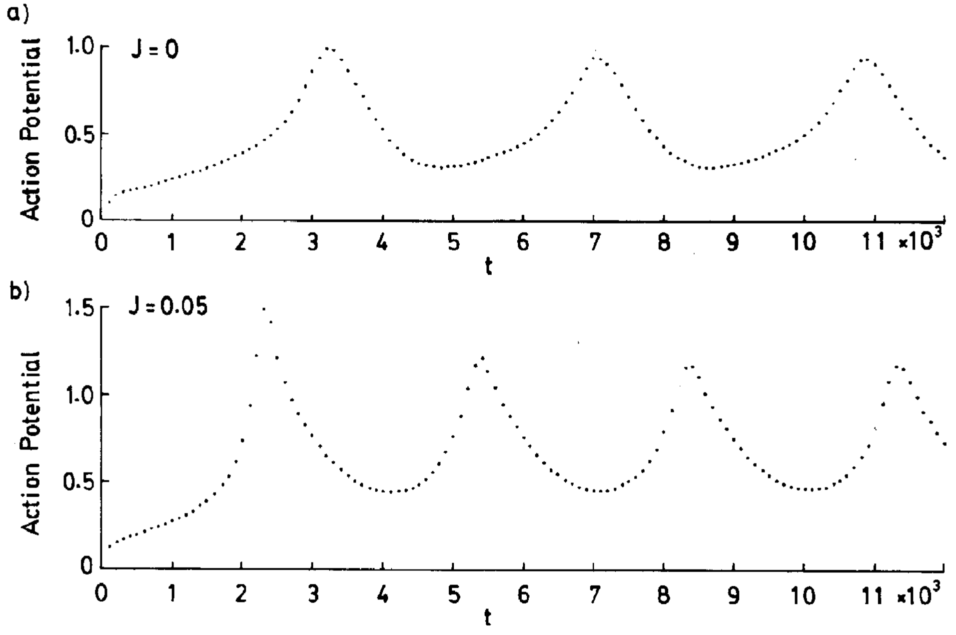

Fig. 4. Self-sustained oscillation of action potential when the ion concentration of the A side solution is higher and that of the B side solution lower than the threshold. (a) No current flowing, (b) current flowing from the A to B side. The simulation of the current effect is done by adding J to the right-hand side of eq. 8 and subtracting the same value from eq. 9. In eqs. 6–9, the following parameters were chosen: $X_{A0} = 3.5$, $X_{B0} = 10^{-4}$, $D_M = 1.1$, $D_A = 0.2$ and $D_B = 2$.

where r is the substantial resistance across the interface. The action potential V_{AB} defined by the potential of the A solution with respect to the B solution is given in the form,

$$V_{AB} = (I_A + I_B)r. \tag{27}$$

In each time step of the calculation, X_A, X_B, Y_A and Y_B are determined from eqs. 6–9, I_A and I_B by eqs. 21 and 22 and finally the forward and reverse rate constants in the next step are obtained from eqs. 23–26. In the next step, the values of the rate constants thus obtained are substituted into eqs. 6–9 and the process is iterated. Fig. 4a shows an example of the time dependence of the action potential obtained from eq. 27 in each time step, taking the substantial resistance of the interface r to be constant.

We can see that the spontaneous oscillation of the potential is of the intermittent type. An interpretation of this behavior is as follows; if the ion concentration X_A near the gate of the membrane channel in the A side solution exceeds some threshold value, the reaction rate for X ion adsorption by the membrane rapidly increases and the concentration of the intermediate substance increases from P_2 to R_3 in fig. 2, however, the high-rate adsorbing reaction will exhaust the X_A ion near the membrane and reduce its concentration to a lower value below threshold, and then the adsorbing reaction rate returns to the usual level. This is one shot of firing. After that, the concentration of X_A ion gradually increases through diffusion from the start, X_A exceeds the threshold value, and then the next firing will occur.

In order to determine the effect of d.c. electric current which was passed through the membrane from the more concentrated to dilute side, we added a current J to the right-hand side of eq. 8 and subtracted the same current from eq. 9. The result is shown in fig. 4b where the oscillation is seen to be rather continuous. An electric current flowing from the A to B side aids the X ion in drifting toward the membrane surface from the start of the reaction, so that the period of oscillation will be shorter.

5. Discussion

A critical change in the conductance of an artificial membrane observed at a threshold concentration of the environmental salt solution has been interpreted by means an autocatalytic model concerning the adsorptive reactions at the gate of each pore of the membrane. Depending on the salt concentration in the solution, two stable states for the membrane appear, i.e., low conductive state and high conductive state. If the ion concentration in the solution of one side of the membrane is kept below the above-mentioned threshold, and that of the other side above, a self-sustained oscillation of the intermittent type is found to occur for the ion concentration near the membrane surface in the solution, the concentration of the intermediate substance in the pore and the electric potential across the membrane.

The autocatalytic reaction model proposed here is of the simple type, in which an intermediate product MX serves as a catalyst for the adsorption of another ion X and which is expressed by eq. 2. In actual membrane systems, however, many molecules probably take part in an autocatalytic reaction as expressed by eq. 3 and the transition between the two states may be connected with the phase transition of the molecular assembly.

In the present model, the action potential across the membrane was defined by the potential drop due to the ionic current through the interface between the membrane and the solutions; the interface was supposed to have a high resistance compared with the solution and the inner part of the membrane.

An alternative view of the action potential is that it arises from an extra charge emerging from the interface as a result of the ionic current through the membrane. In this case, the polarity of the action potential will be the opposite of that for the potential drop. The former corresponds to a view regarding the membrane as a resistor, whereas the latter considers the membrane as a condenser. Both effects are considered to be included in the action potential of actual membrane systems and experiments on the polarity of the action potential are desirable to ascertain which effect is predominant for each kind of artificial membrane system.

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